



Workshop Session

ACLE 3  
OCTOBER 2008

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Luther College (Victoria)

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# *Beyond Skills:* *tinkering to thinking*

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*Moving into Generalising and Modelling*

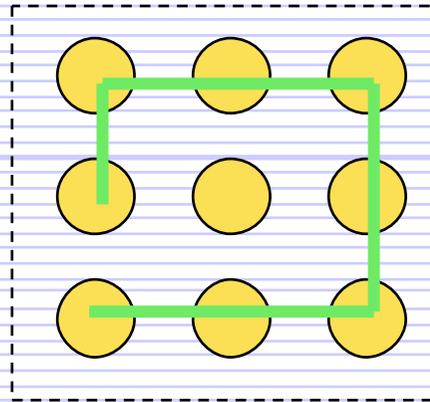
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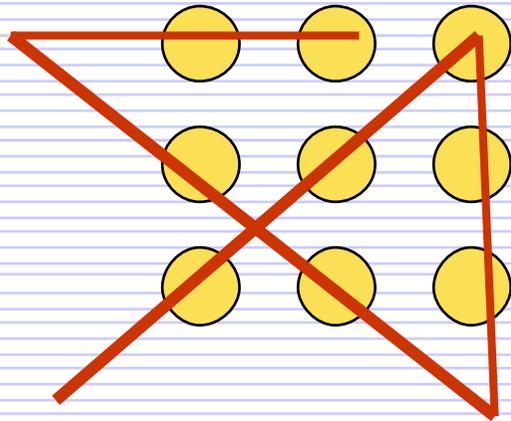
# Boxed in or breaking out ?

How can you pass through all points with 4 connected straight lines?



# Does BOB win ?

How can you pass through all points with 4 connected straight lines?

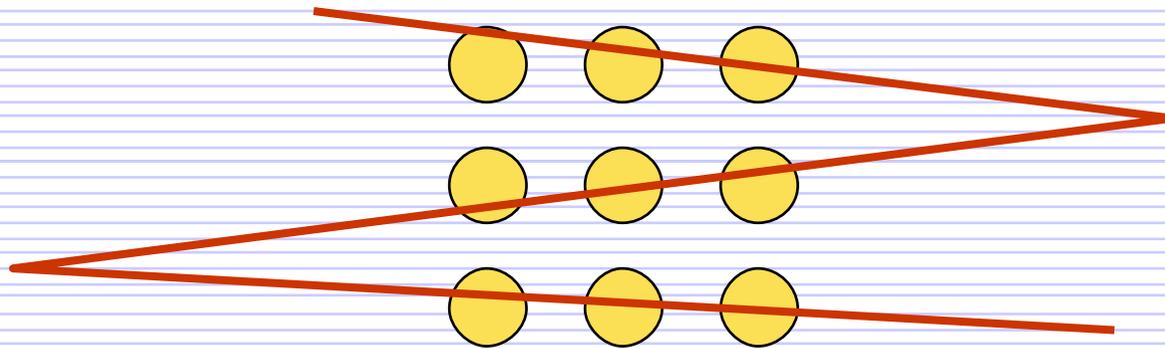


*A solution is not the  
end of a problem*

*can you do it with 3  
lines?*

# Is this the solution?

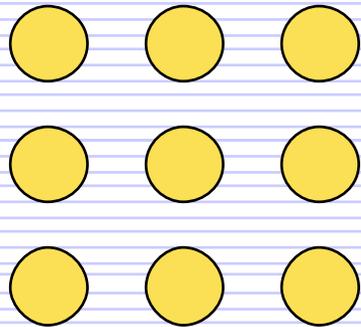
How can you pass through all points?



*Can you do it  
with one line?*

# Is this the solution?

How can you pass through all points?

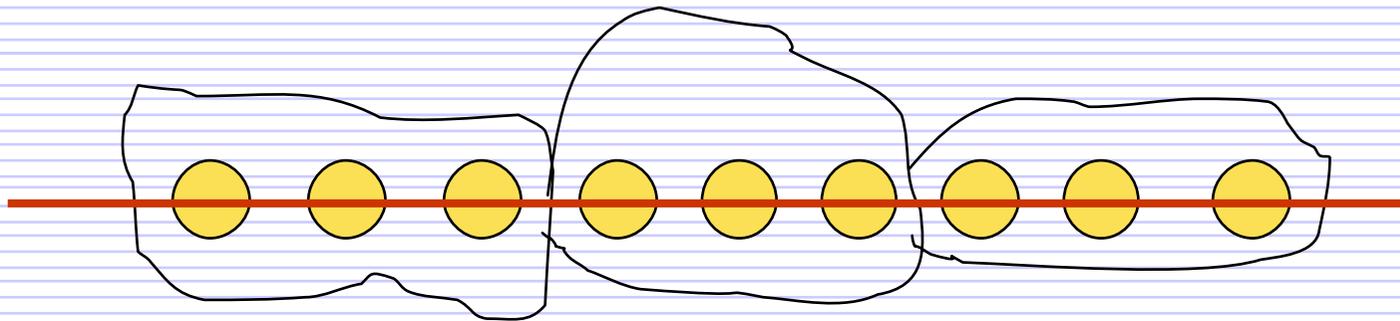


*Can you do it  
with one line?*

# SA Friday?

## Pushing the constraints

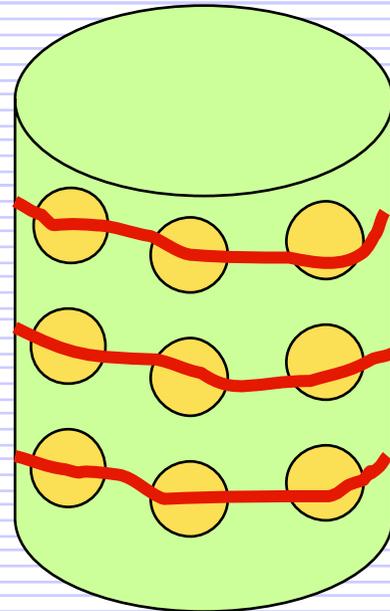
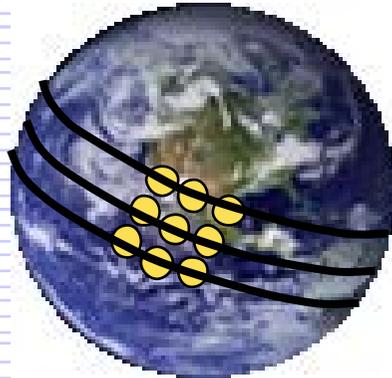
How can you pass through all points?



# The problem is

Do we teach mathematical modelling or curricular mathematics?

How can you pass through all points?



# ***RELEVANCE***

**When am I every  
going to use this?**

Mathematics lessons are very often not about anything. You collect like terms, or learn the laws of indices, with **no perception why** anyone needs to do such things

COCKROFT 'Maths Counts' 1982

**Math skills**

**Maths kills**

## A little boy and his transformers

Which one is the most powerful?

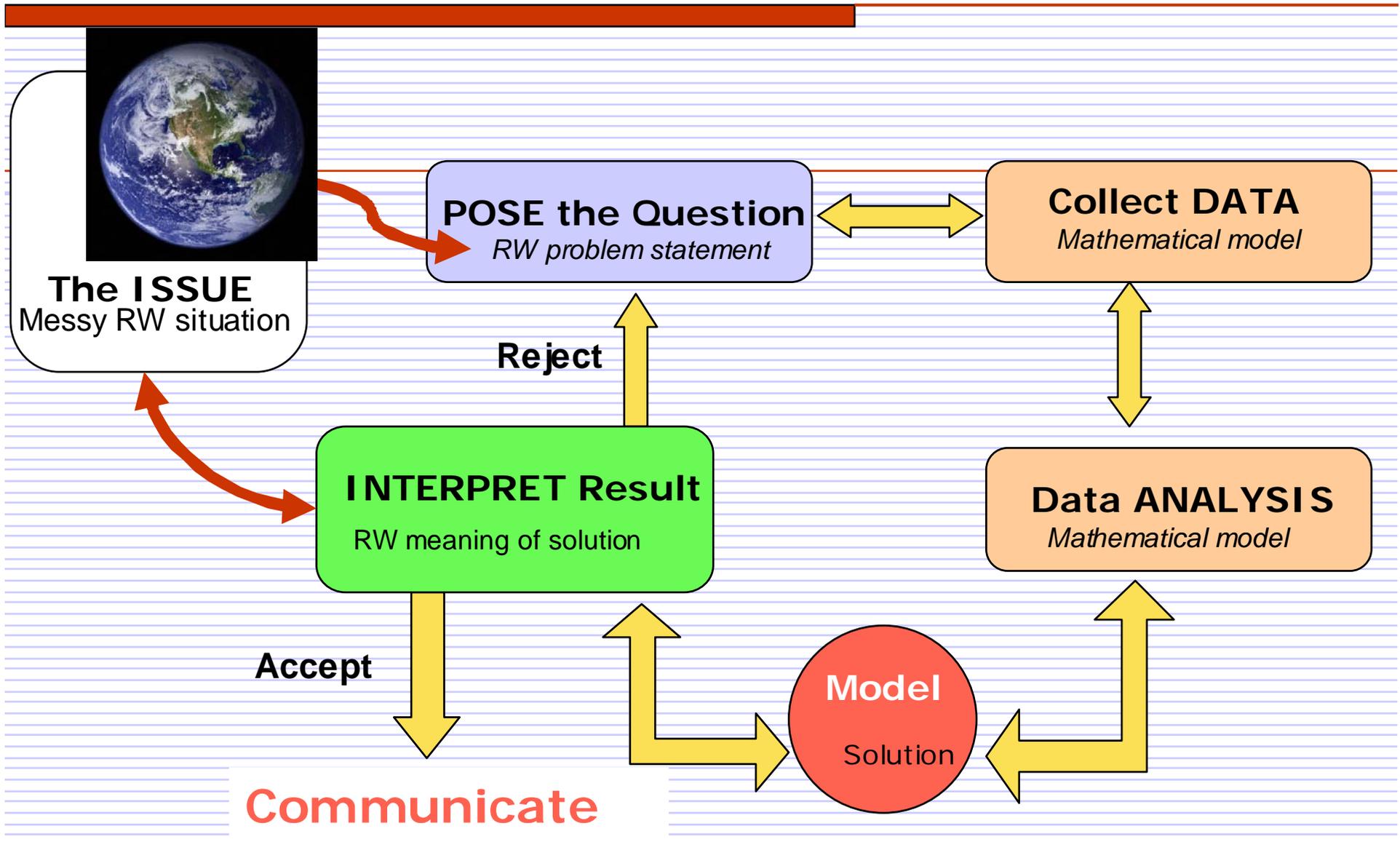
Overbite 50.5

Snarl 51

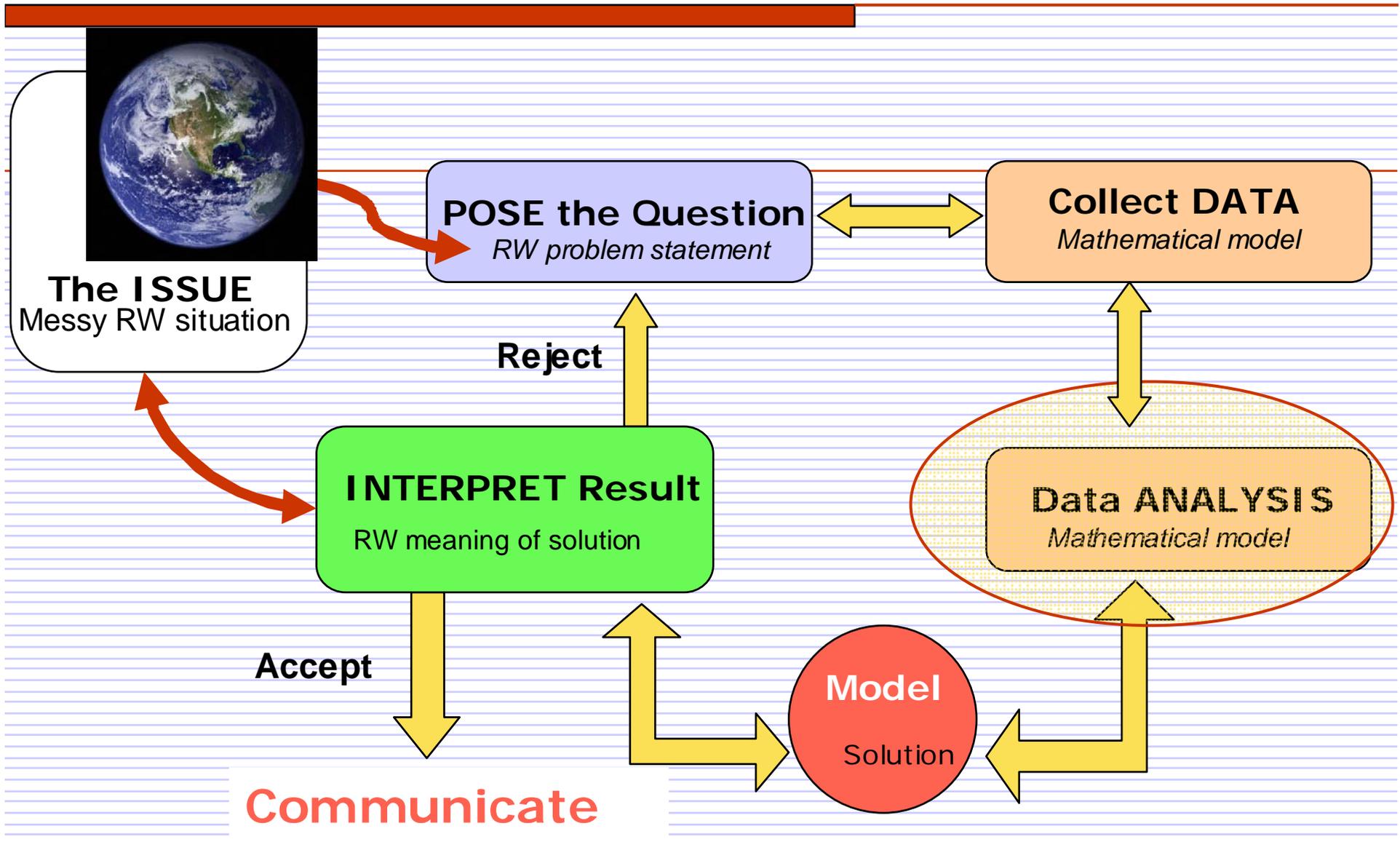
Dreadwing 49

using technology to scaffold a problem

# Modelling Cycle



# Modelling Cycle





# Einstein

an observation on the nature of the way in which knowledge grows

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*'If we always do what we have always done, then we will always get what we have always gotten.'*

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# *Generalisation* – not if but when

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There is a stage in the curriculum when the introduction of algebra may make simple things hard, but **not teaching Algebra will soon render it impossible to make hard things simple**

# Change for \$10

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**Excuse me Mister can you give me change for \$10.**

**With a pocket full of \$1 and \$2 coins ...**

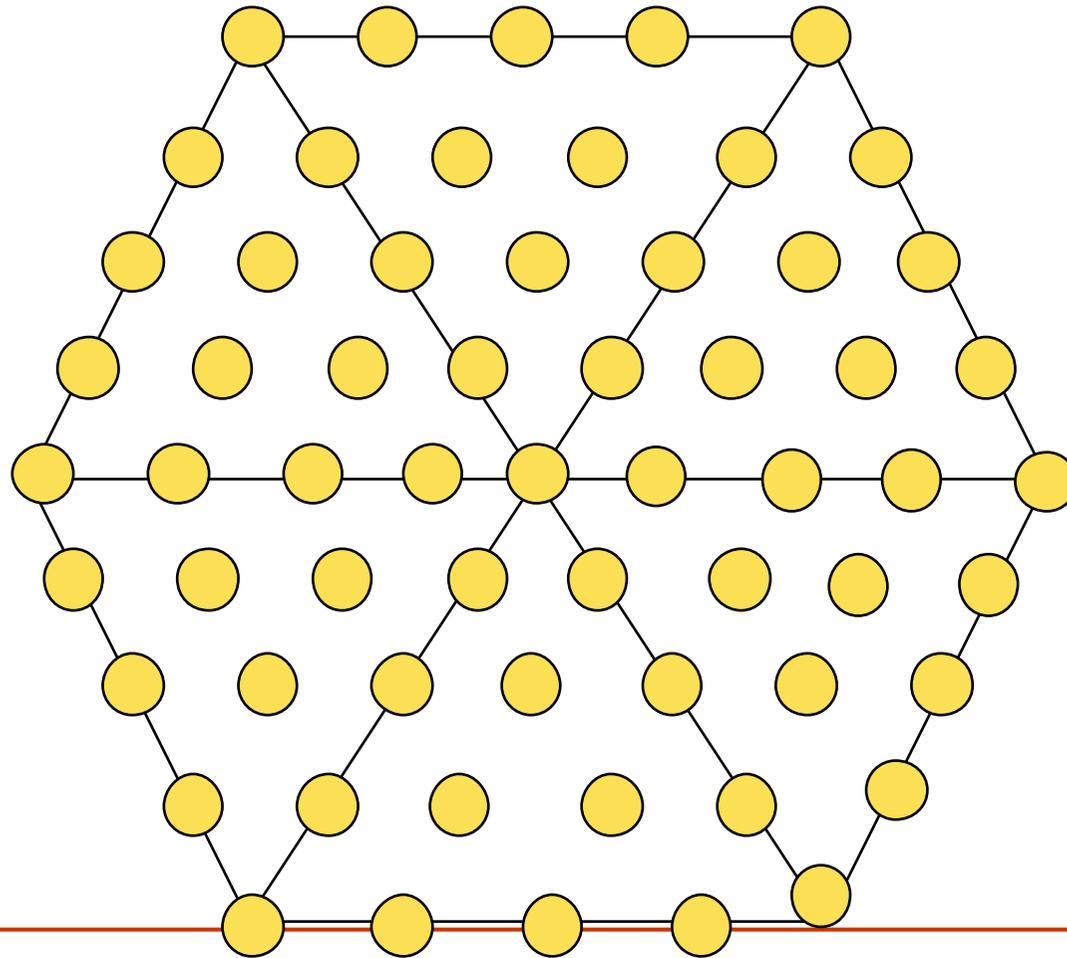
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# Change for \$10

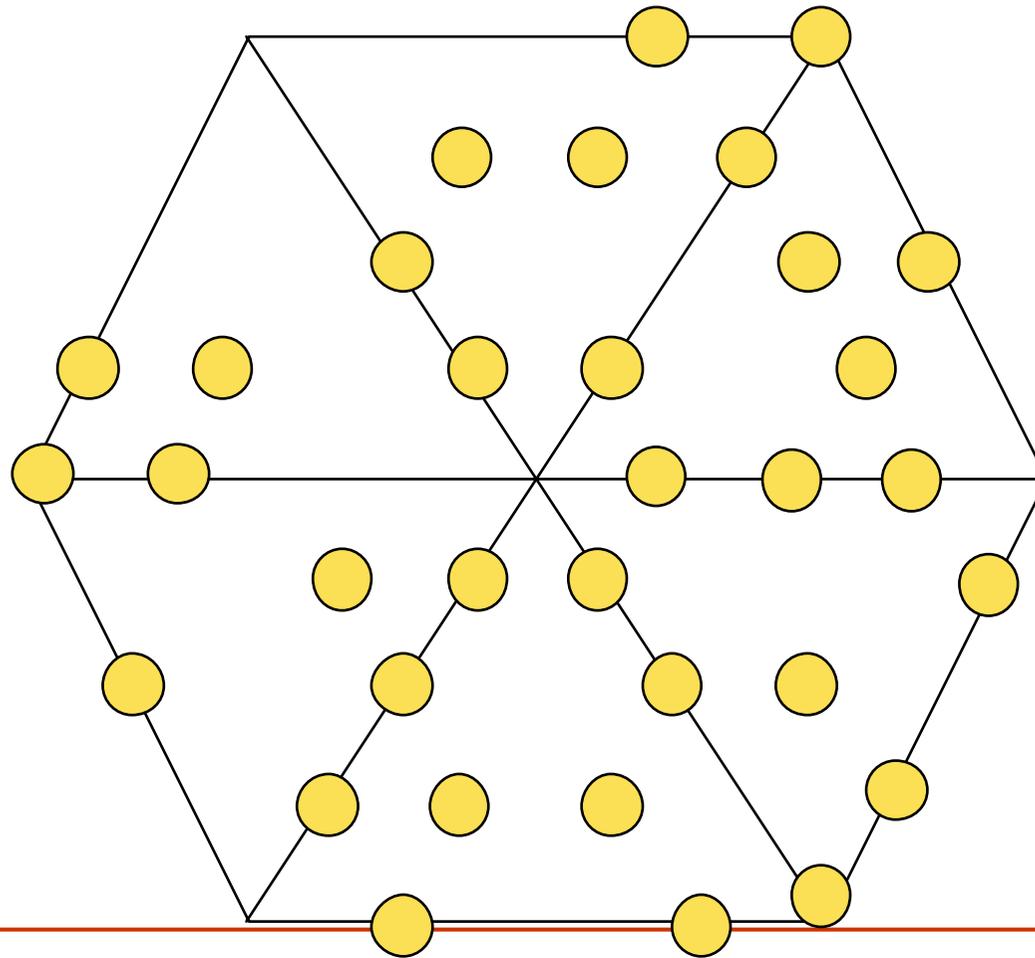
	A value	B ways	C ttl
•			
1	1	1	231
2	2	2	
3	3	3	
4	4	5	
5	5	8	
6	6	13	
7	7	21	
8	8	34	
9	9	55	
10	10	89	

*Generalisation*

# HEXAGON

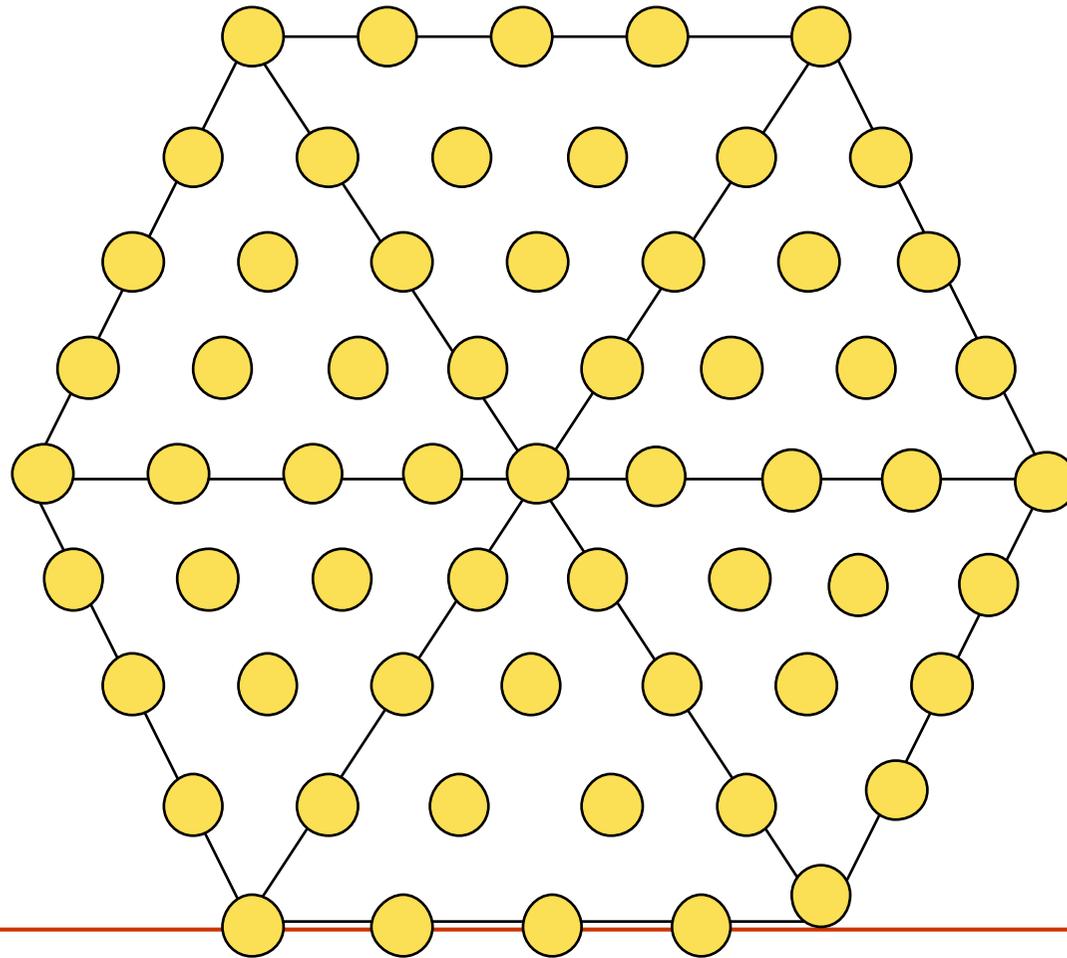


# HEXAGON

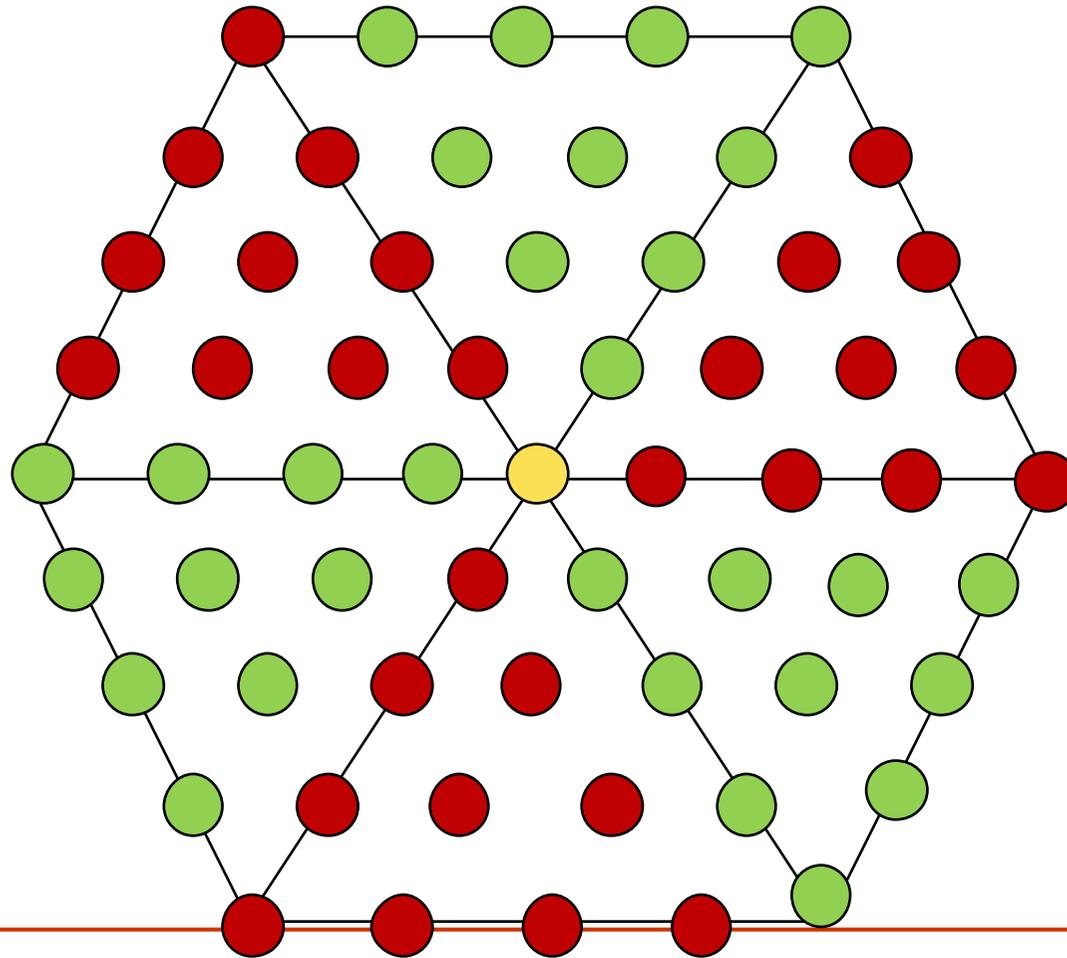


*What percentage have have been removed?*

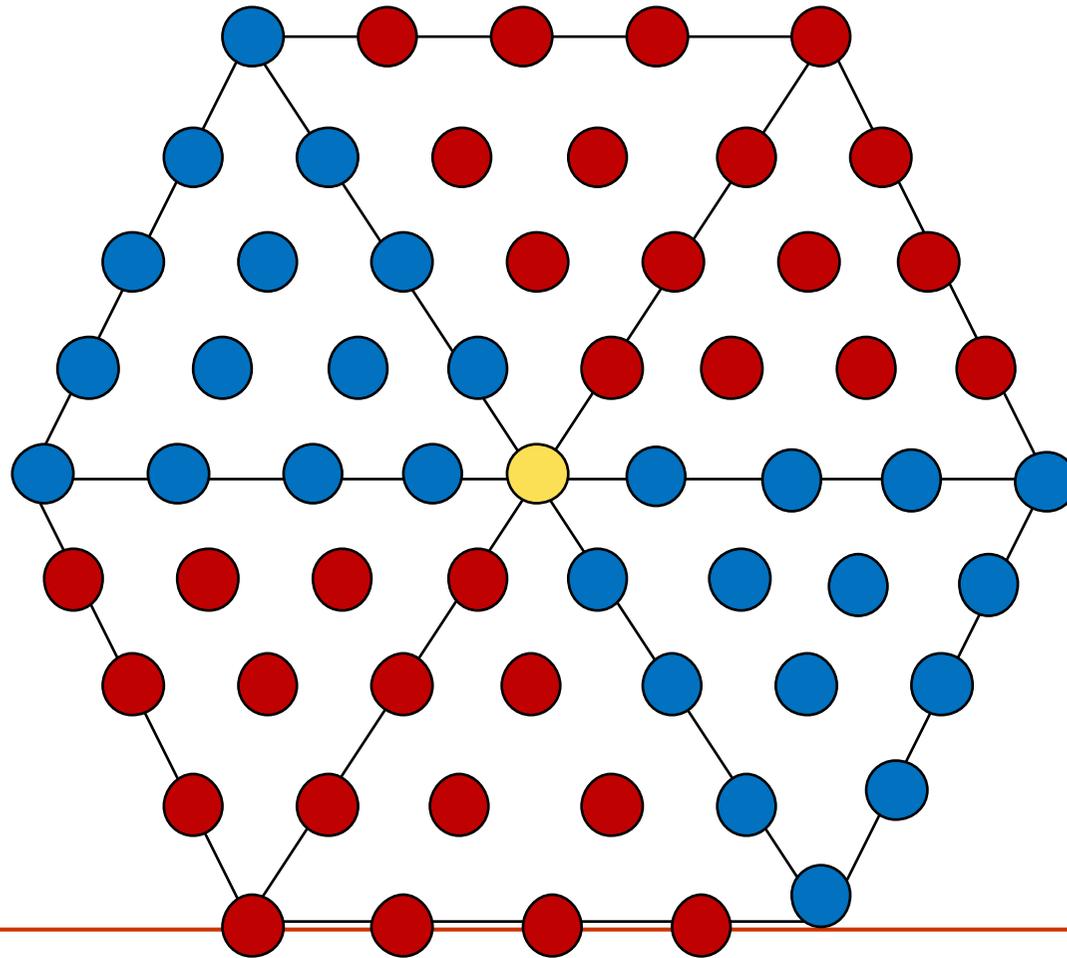
# HEXAGON



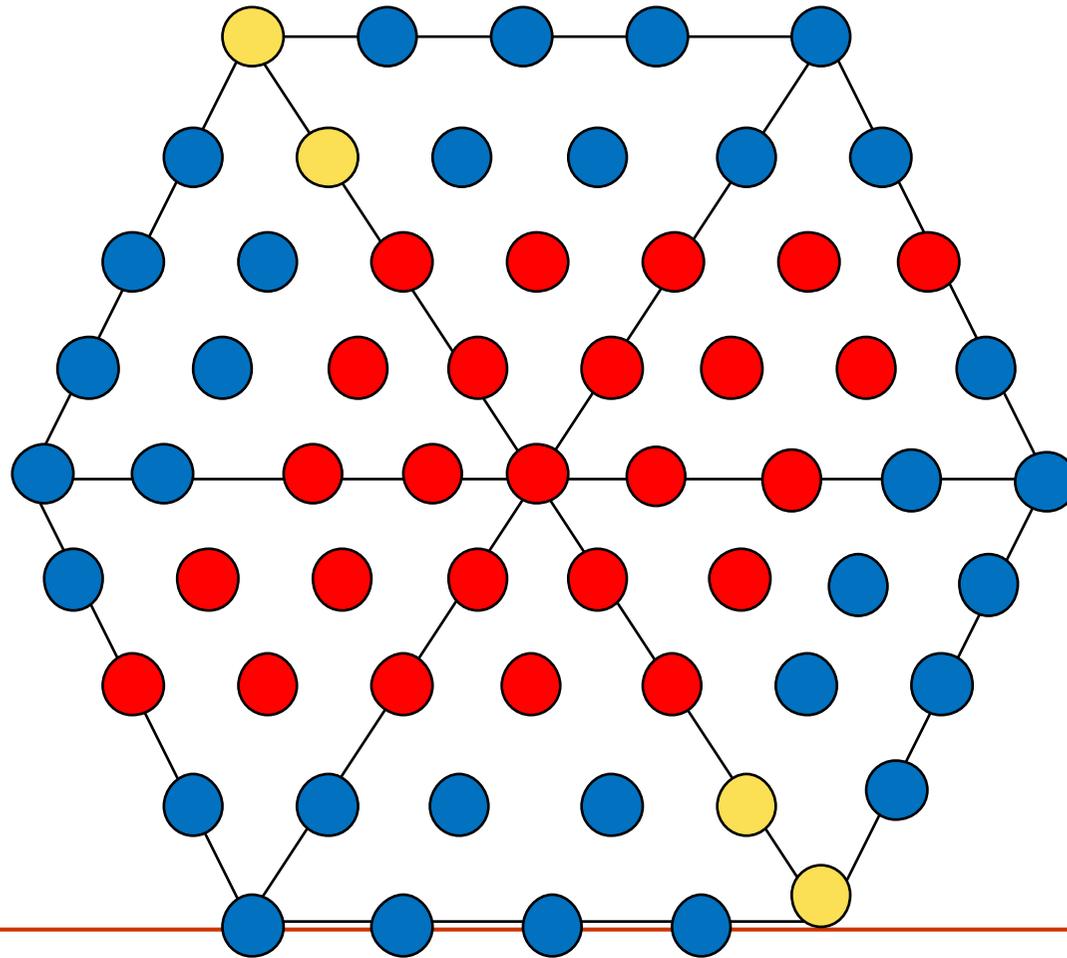
# HEXAGON



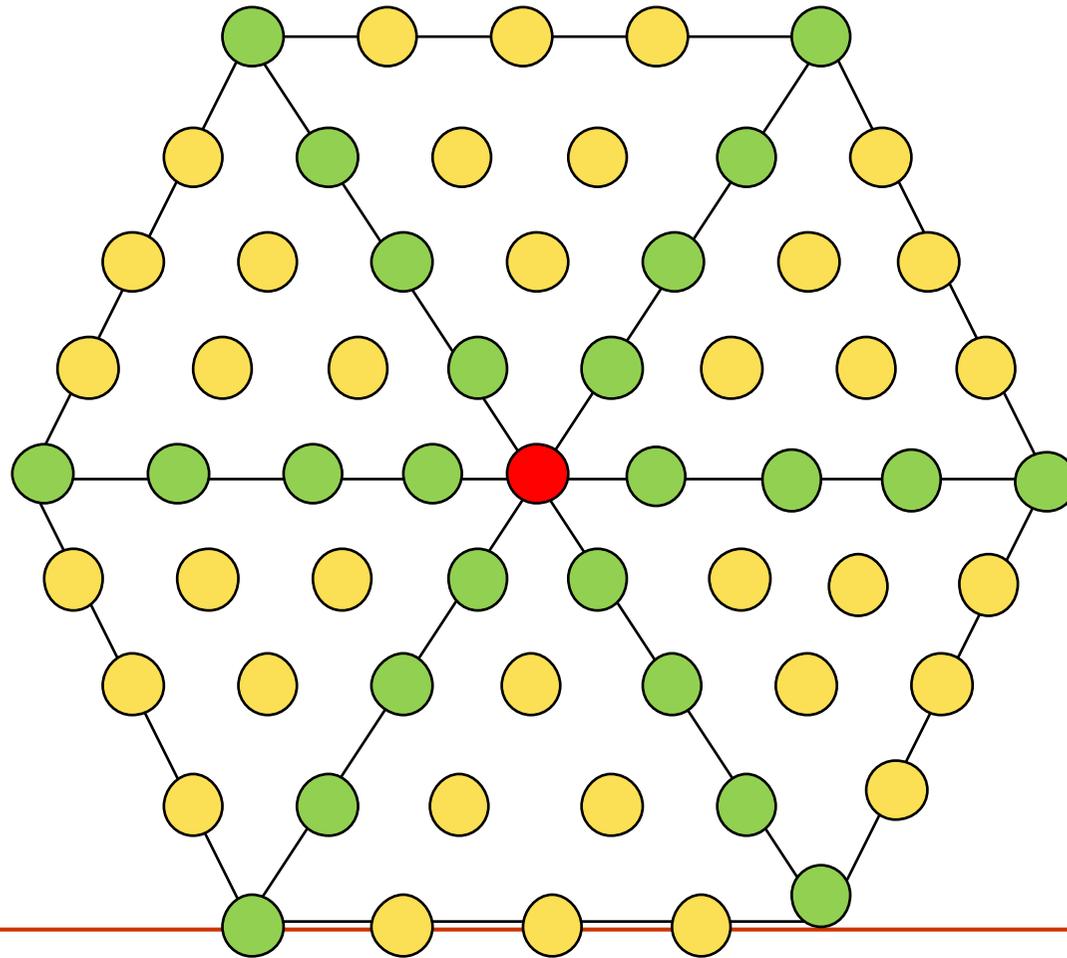
# HEXAGON



# HEXAGON



# HEXAGON



# HEXAGON

$\frac{6 \cdot (n-1)}{2} \cdot (1+n-1) + 1$	$3 \cdot n^2 - 3 \cdot n + 1$
$1 + 2 \cdot (n-1)^2 + \frac{2 \cdot (n-1)}{2} \cdot (n+2)$	$3 \cdot n^2 - 3 \cdot n + 1$
$n^2 + 2 \cdot (n-3) + 4 \cdot 2 \cdot (n-1)$	$n^2 + 10 \cdot n - 14$
$1 + 6 \cdot (n-1) + 3 \cdot (n-2) \cdot (n-1)$	$3 \cdot n^2 - 3 \cdot n + 1$
$3 \cdot n^2 - 3 \cdot n + 1   n=5$	61
$n^2 + 10 \cdot n - 14   n=5$	61
$3 \cdot n^2 - 3 \cdot n + 1   n=6$	91
$n^2 + 10 \cdot n - 14   n=6$	82

*Which square number ends in 3?*

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**625**

*The product of two whole numbers separated by one whole number is one less than a square whole number*

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# Knot the rope

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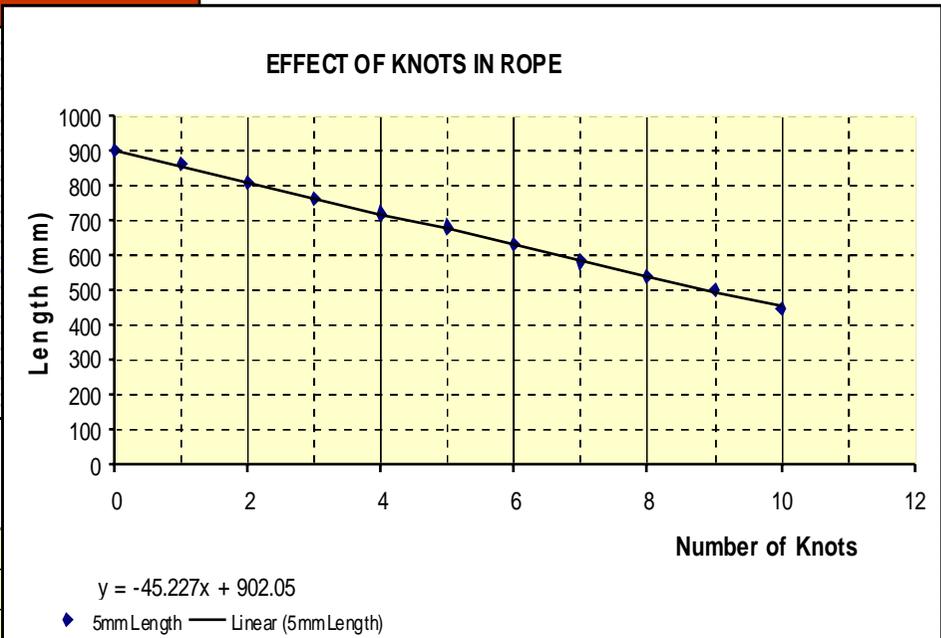
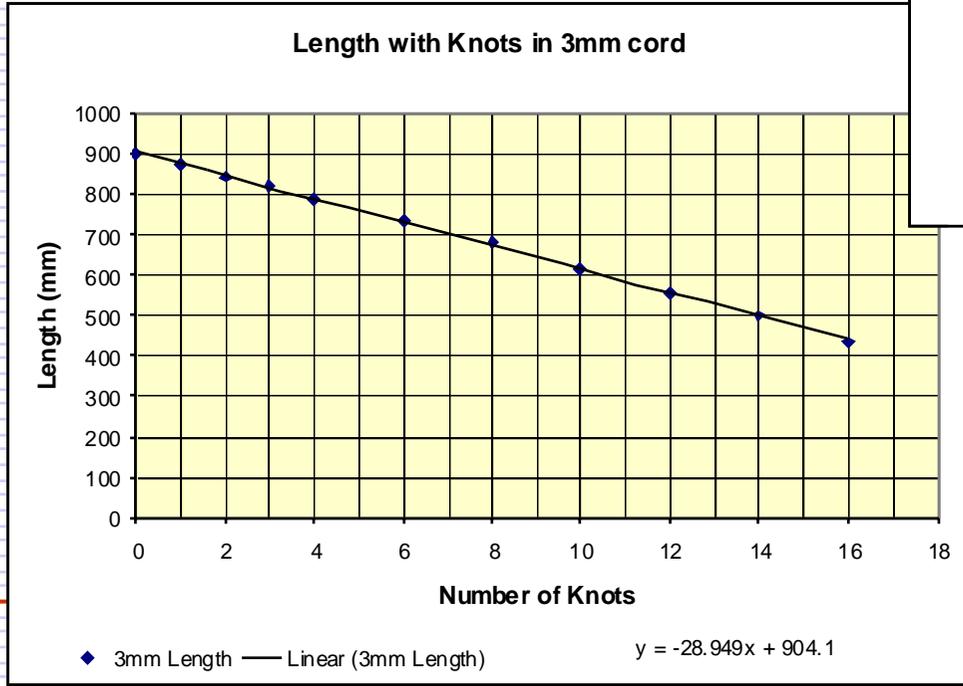
**How long is the piece of rope?**

**How much rope is required to tie a knot?**

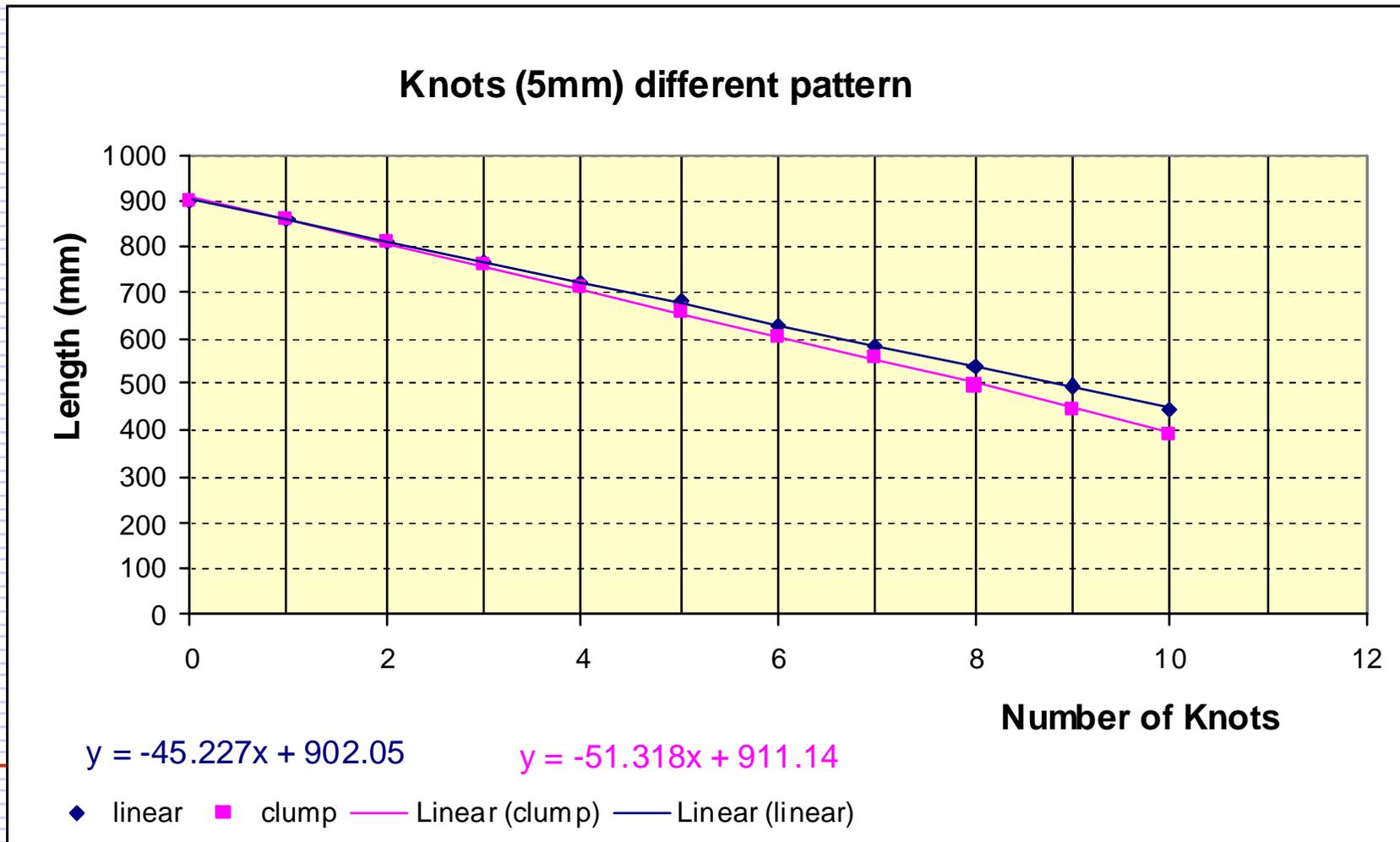
**How many knots can you put into the rope?**

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# Knot the rope

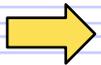
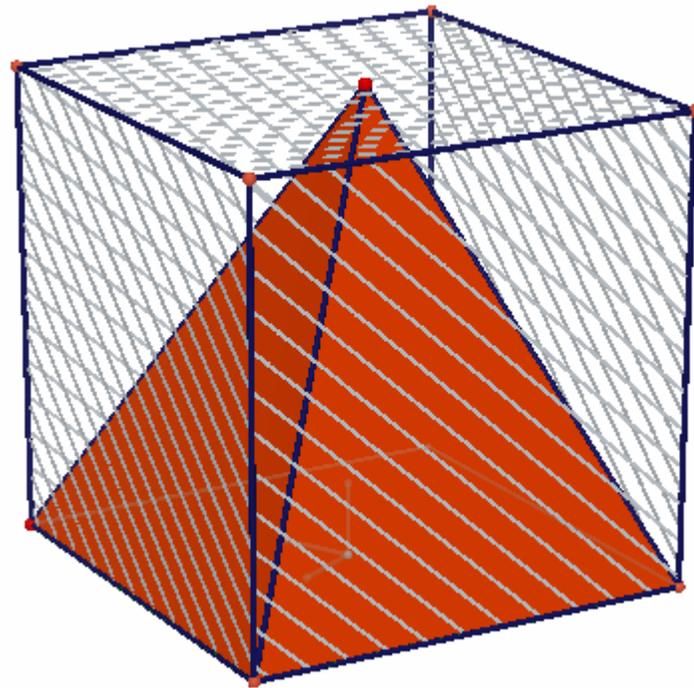


# Knot the rope



# Pyramid Volume

Why  $1/3$  ?



# What features of a mathematics classroom **make a difference** in what students ultimately learn?

Highest learning gains on a mathematics performance assessment were related to the **extent to which tasks were set up and implemented in ways that engaged students in high levels of cognitive thinking** and reasoning.”

Smith & Stein 1998

# How can I support High-Level Thinking and Reasoning

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- ❑ Tasks that build on students' existing knowledge.
- ❑ a sustained press for justifications, explanations, and meaning through questioning, comments, and feedback.
- ❑ Frequently drawing together the conceptual connections.

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connection + dialogue + reflection

# How can I support High-Level Thinking and Reasoning

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- ❑ Scaffolding a task while maintaining the complexity.
- ❑ The teacher or students model high-level performance.
- ❑ Giving students the means to monitor their own progress.
- ❑ Allowing students sufficient time to explore, not too little or too much.

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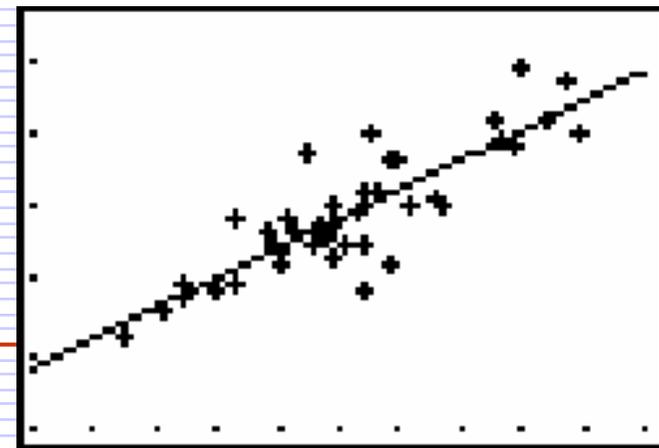
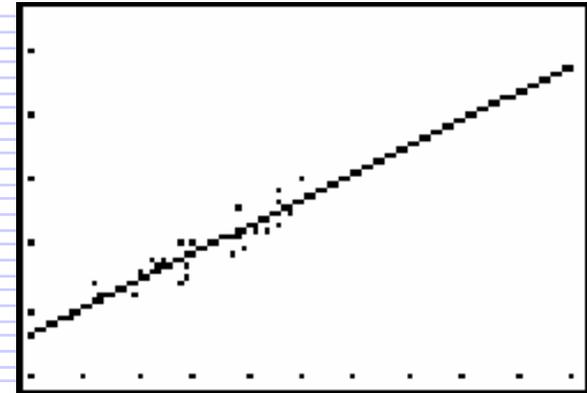
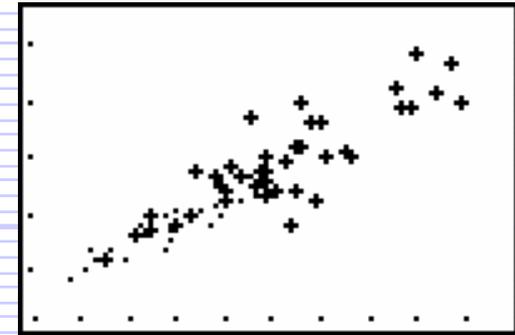
assistance + vision + feedback + TIME

*from application tasks*



*to modelling tasks*

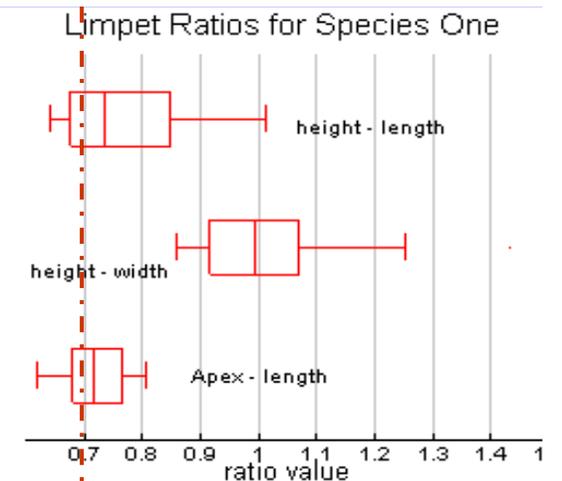
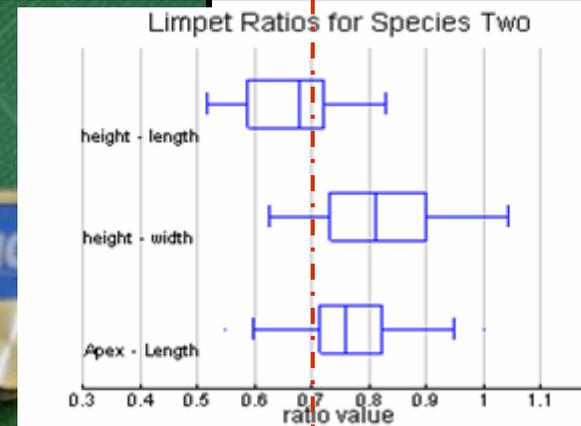
# - the limpet -



# - the limpet -



ratio	Yallingup WA Group 1	Yallingup WA Group 2
Number of shells in sample (n)	28	40
Mean	0.717	0.767
Standard Deviation	0.053	0.088
Lowest ratio (Xmin)	0.619	0.548
First Quartile (Q <sub>1</sub> )	0.680	0.714
Median	0.715	0.759
Third Quartile (Q <sub>3</sub> )	0.765	0.823
Highest Ratio (Xmax)	0.807	1.000
Interquartile Range	0.085	0.109



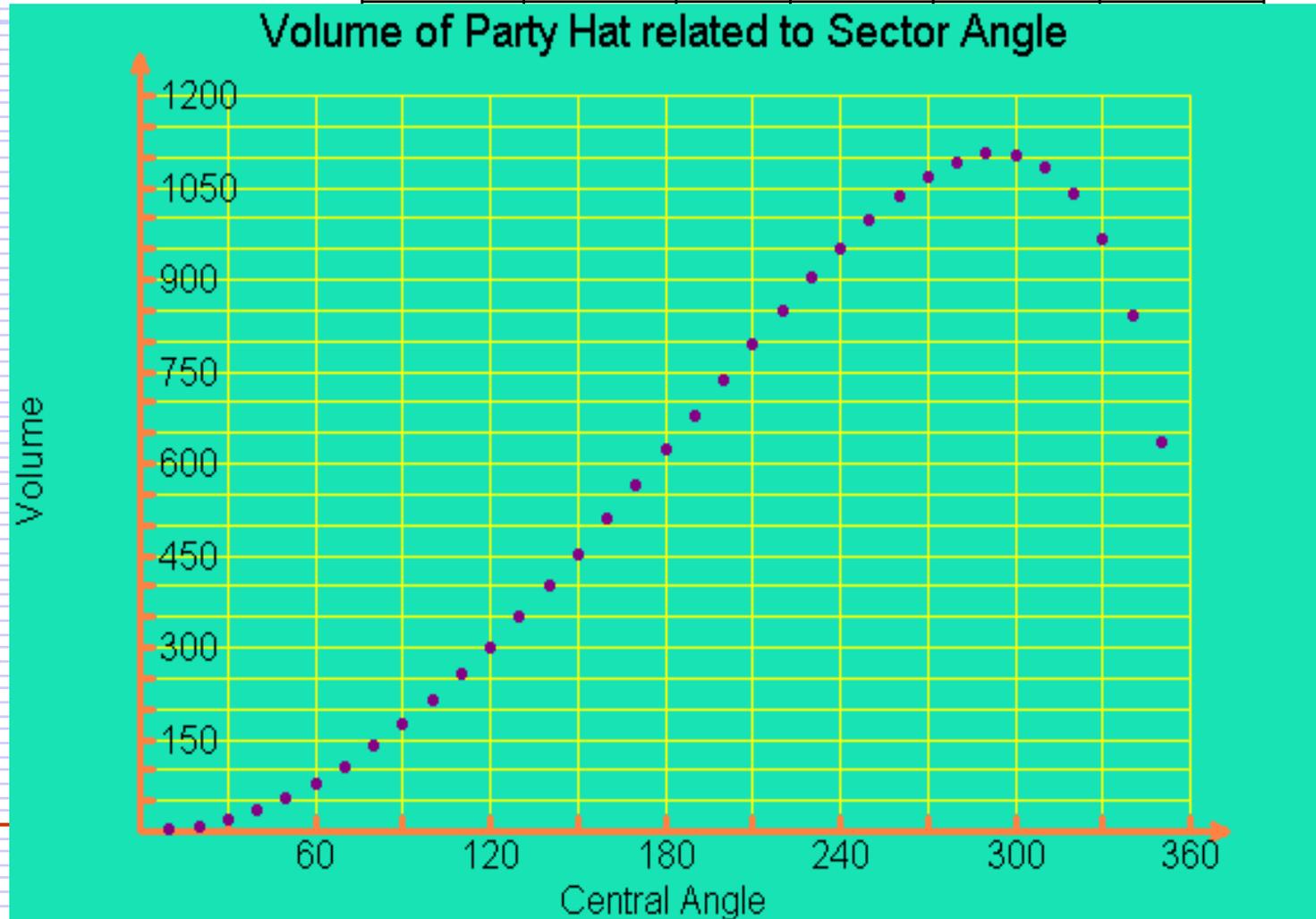
# - the limpet -



Central Angle	SECTOR		CONE		
	Arc Length	Area	Base radius	Cone Height	Volume
345	72.3	433.54	11.5	3.43	474.73
330	69.1	414.69	11.0	4.80	607.68
315	66.0	395.84	10.5	5.81	670.72
<b>300</b>	<b>62.8</b>	<b>376.99</b>	<b>10.0</b>	<b>6.63</b>	<b>694.63</b>



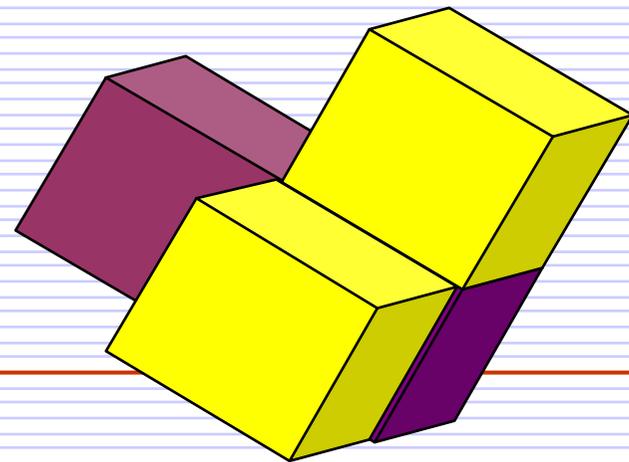
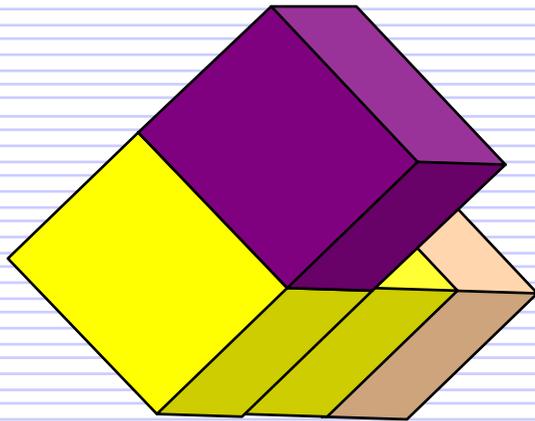
Fig. 2. Perspective drawings of (A) the less asymmetrical cone (asymmetric 1 in Table 1) and (B) the more asymmetrical cone (asymmetric 2 in Table 1) used in these experiments.



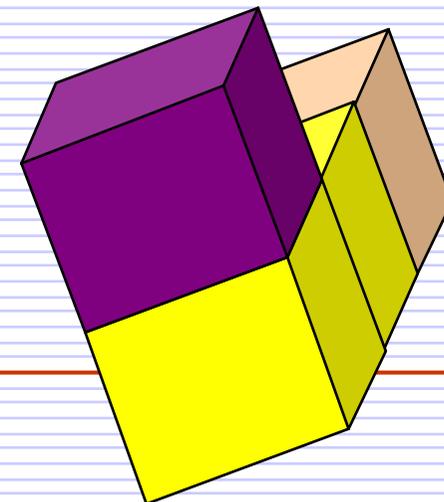
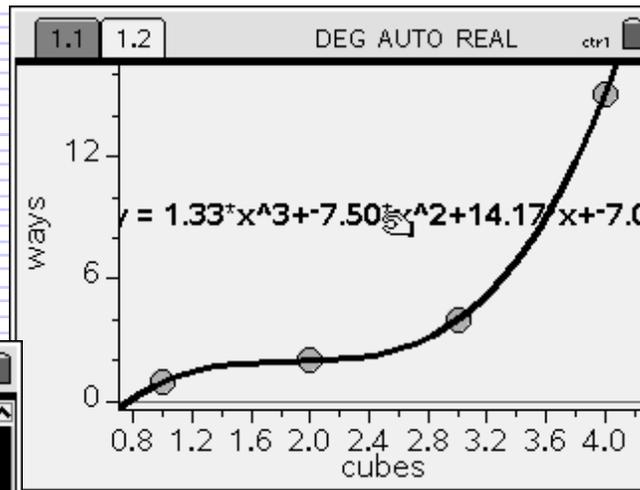
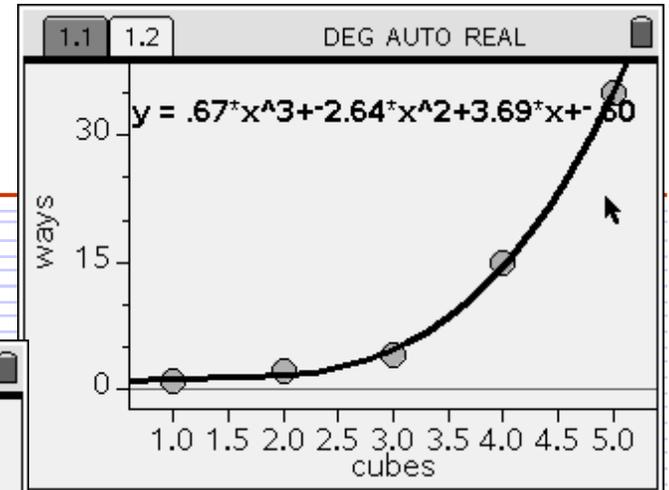
# CUBE HOUSES

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How many 4 cube houses are there?



# 5 CUBE HOUSES



DEG AUTO REAL

	A <sub>c</sub>	B <sub>w</sub>	C	D	E	F	G
		=4/3*c					
1	1	1.	1				
2	2	2.	2				
3	3	4.	3				
4	4	15.	4				
5	5	43.	5				

C7 | 1

# Decartes

$$a \cdot x^2 + b \cdot x + c$$

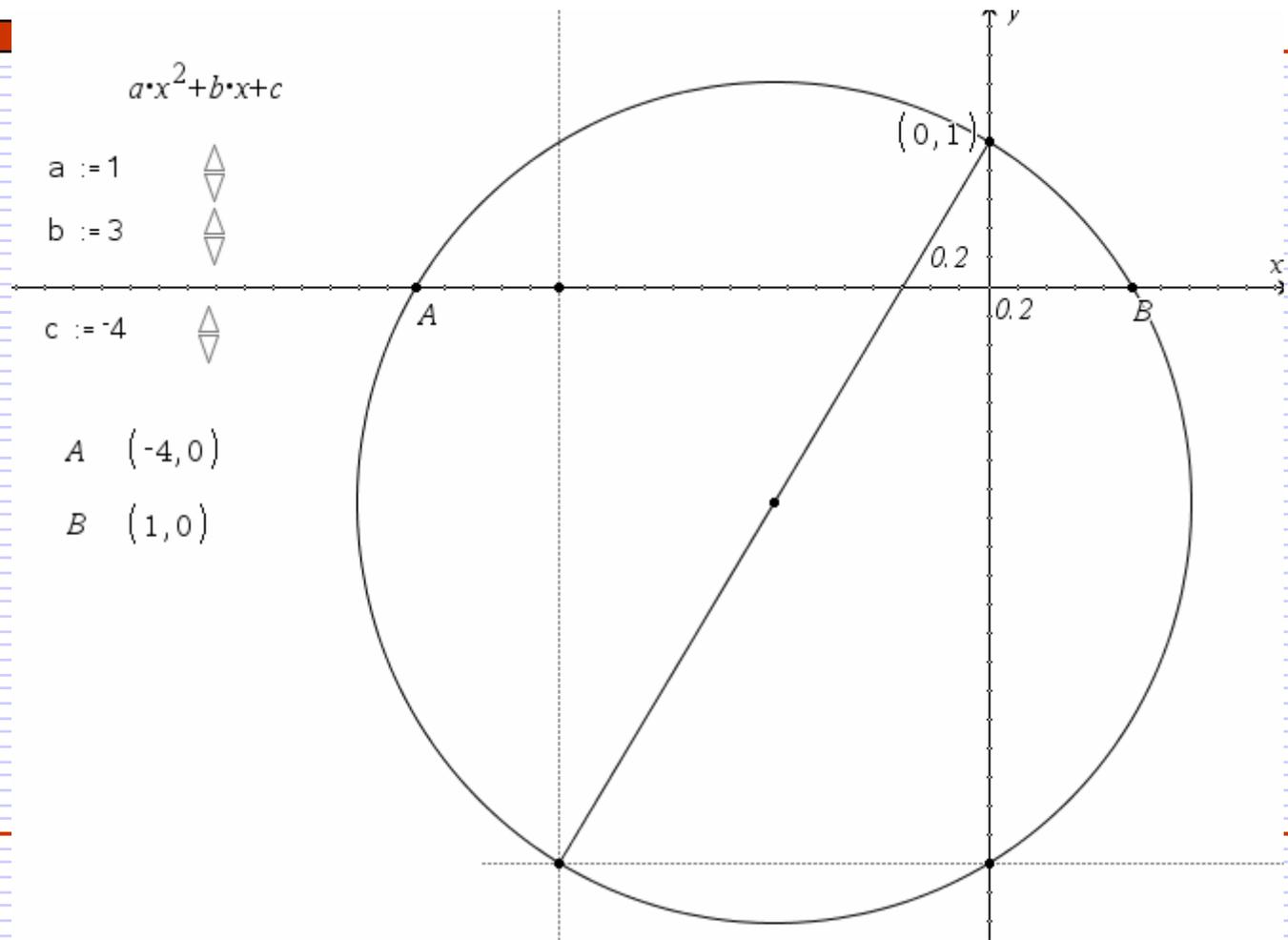
$$a := 1$$

$$b := 3$$

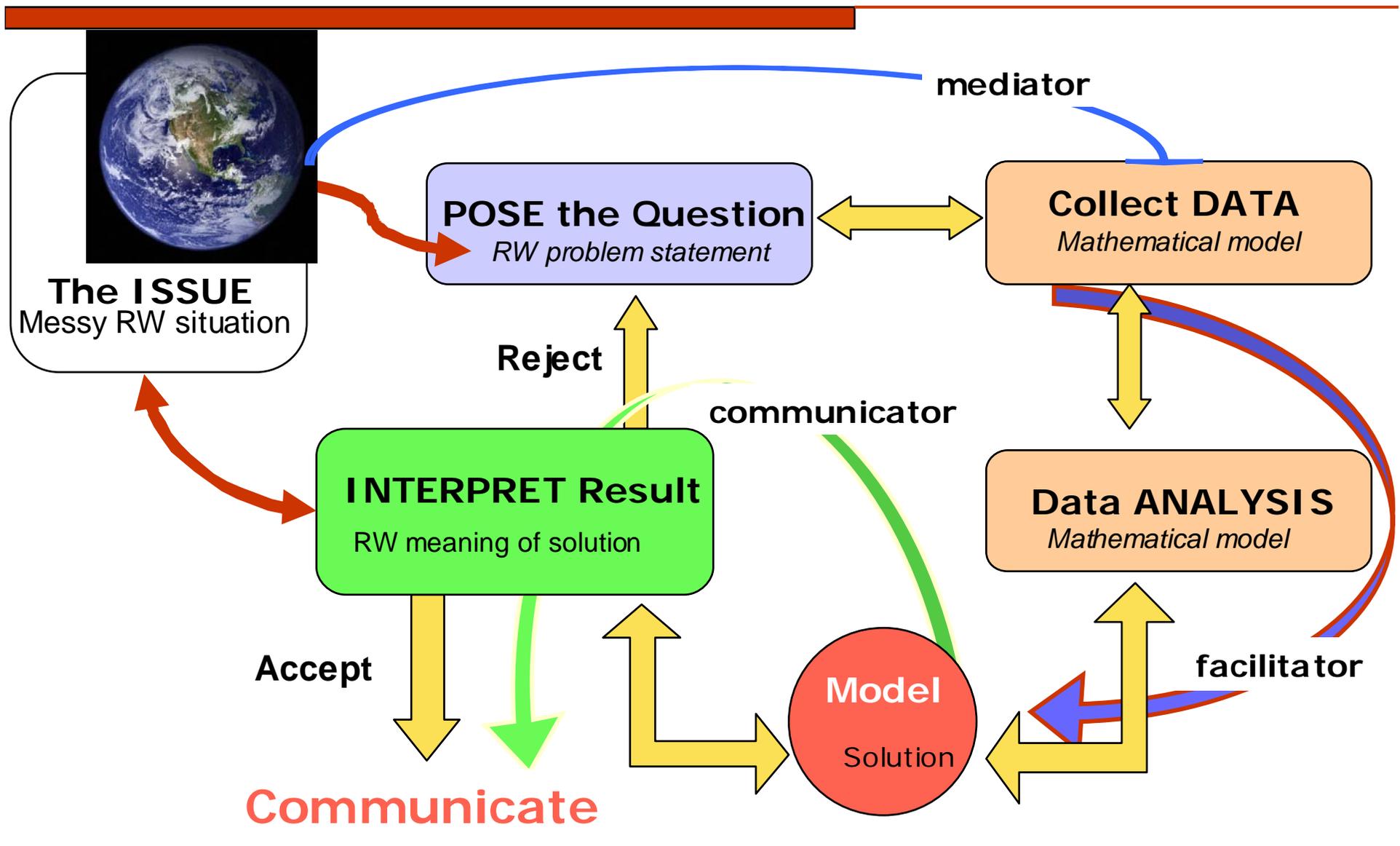
$$c := -4$$

$$A \quad (-4, 0)$$

$$B \quad (1, 0)$$

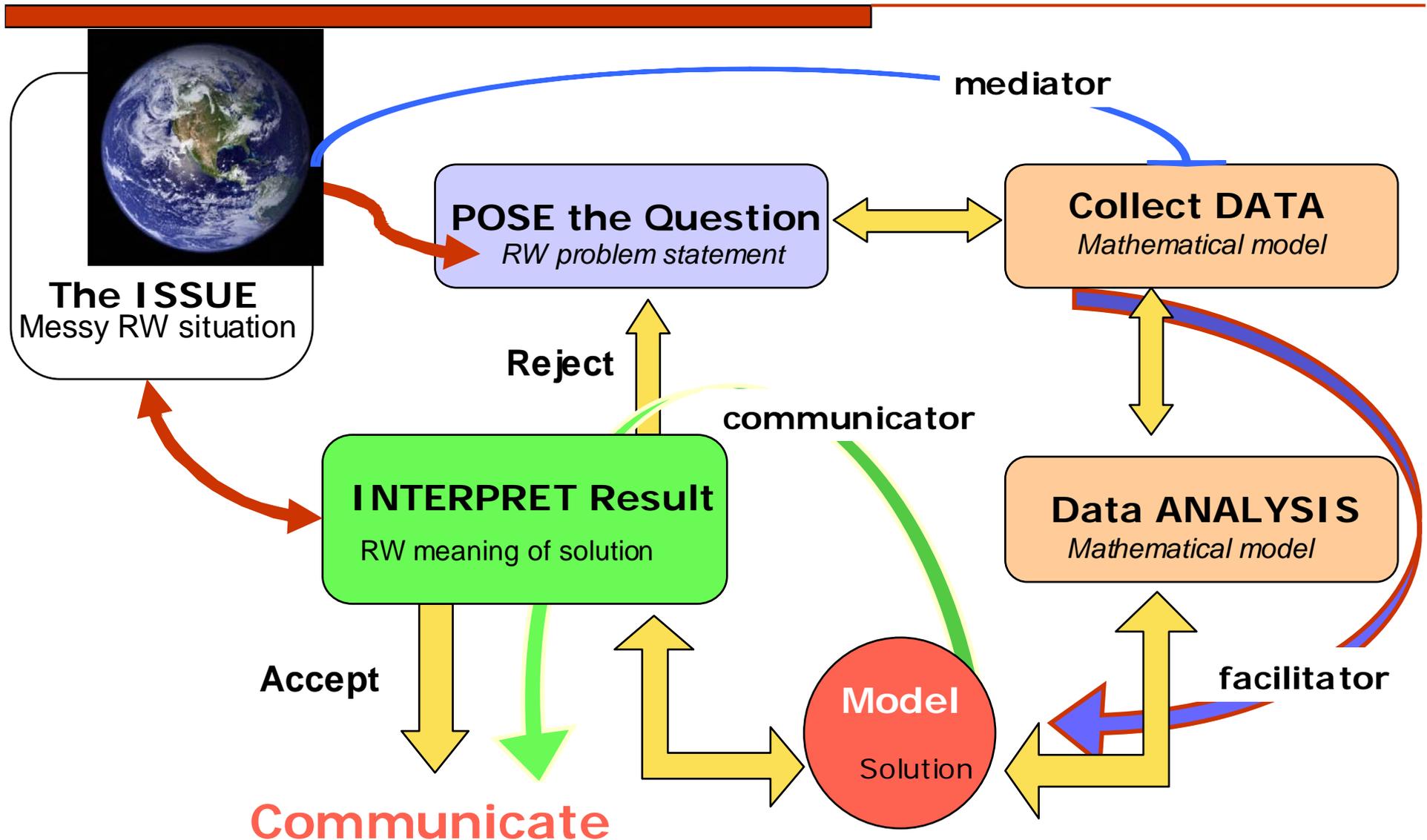


# Modelling Cycle



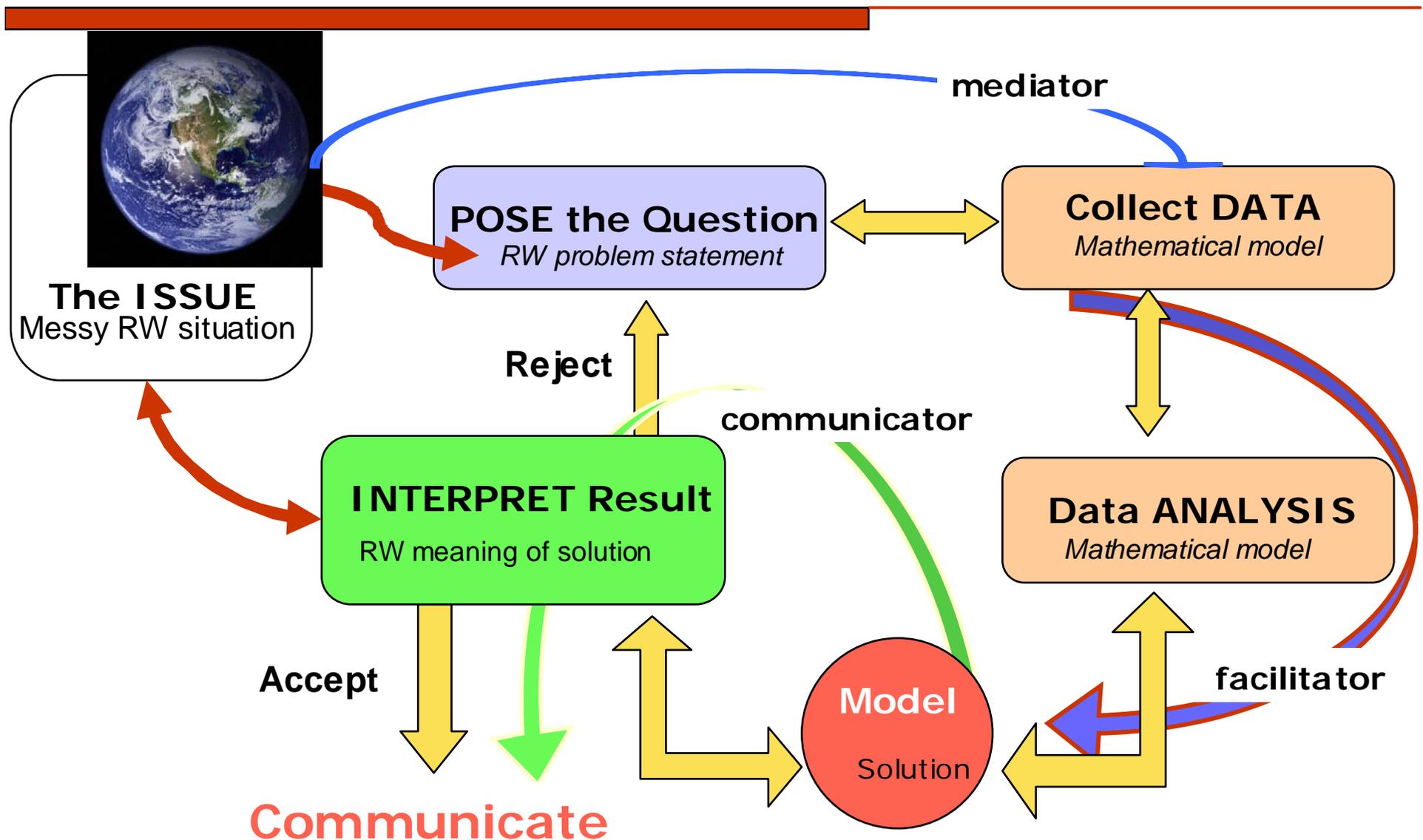
Students **choose, use and develop mathematical models** and procedures with attention to assumptions and constraints

# Modelling Cycle



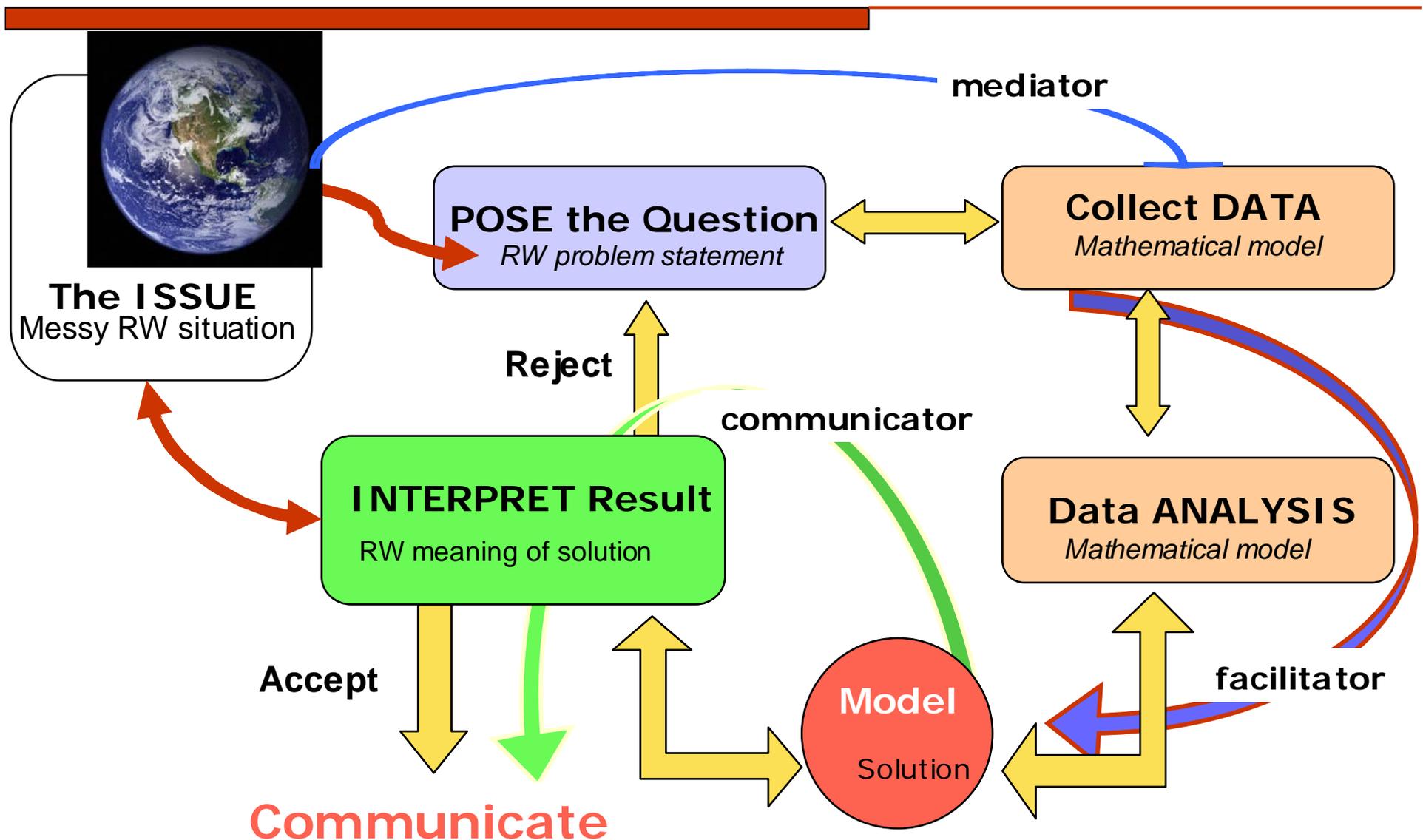
They solve problems in a wide range contexts and **communicate the results** of these investigations. They **extend their problem solutions by generalising, or changing the initial constraints** of a situation for further investigation.

# Modelling Cycle



Students use technology to develop mathematical ideas and solve problems.

# Modelling Cycle



They describe the **major features of mathematical structure**, and use of **logical argument in mathematical discourse and applications of mathematics**.

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If mathematics teachers fill the allotted time with drilling their students in routine operations they *kill their interest*, hamper their intellectual development, *misuse their great opportunity*.

But if they **challenge** the curiosity of their students by setting them problems proportionate to their knowledge, and **help them to solve** their problems with stimulating questions, they may give them a taste for, and some means of **independent thinking**.

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Lame has been approached by TTT Farms (TOMMY TINN TROUT FARMS) to suggest an appropriate management procedure for a potential new fishery in the lake at Lilydale National Park.

When first investigated, a lake in a Lilydale National Park was stocked with approximately 10000 trout. Research experience has shown in similar lakes to that in Lilydale NP when left to natural factors, the trout numbers in the lake increase on average by 20% per year. For the lake's fishery to remain viable there needs to be at least 750 fish in the lake.

Overpopulation of fish in the lake is an issue. Overpopulation of the lake can cause a dramatic fish kill, where up to 95% of the fish stock may die. The carrying capacity of the lake is assumed to be approximately 5 times the current capacity of 10000 fish.

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***Strategy 1.***

Do nothing to disrupt with the normal population control of the fish in the lake. Fishing in the lake shall be on a catch and release basis.

***Strategy 2.***

Fishing will be permitted. The total catch allowed for all fishing licenses issued would be 1800 fish per year.

***Strategy 3.***

Have a single approved fishing contractor who is licensed to remove up to 2500 fish from the lake each season.

***Strategy 4.***

Permit the fish population to reach 25000 fish, then issue and monitor amateur fishing licenses that would maintain fish stock at this level.

---

Eeenee-Meenee Lake is currently stocked with 10 000 trout. It has a maximum capacity of approximately 50 000. Should the population reach this level, up to 95% of the population will die. Research has shown that the population increases by 20% every year and that the lake needs at least 750 fish in it to remain viable.

**The formula I used to find the fish population throughout this report is:**

$$(p + 20\%) - x = p$$

where  $p$ ' is the population of the lake (always starting at 10 000 and increasing / decreasing accordingly) and  $x$ ' is the amount taken out of the lake.

*Condition: If:  $p' < 50000$  Then:  $p_n - 95\%$*

This operation is carried out to find the population for each year. The value of  $p$ ' is shown in the graph for each strategy (data tables are provided on the last page of this report).

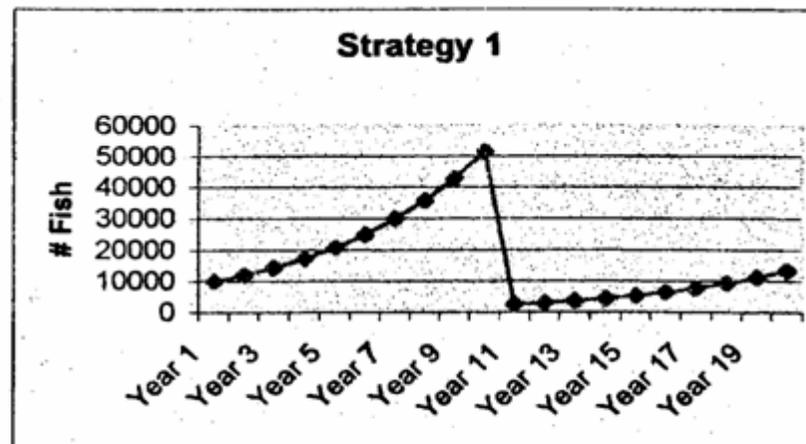
### Strategy 1

**Do nothing to interfere with the normal population of the fish in the lake.**

$$x' = 0$$

After nine years the lake will have exceeded its maximum population of 50 000 by 1597 fish. A fish kill would occur killing 49017 and leaving the lake with a population of 2580 fish. After approximately 16 years the fish population would have exceeded 50 000 again and another 47500 fish would

die. This cycle of reproduction and overpopulation would continue indefinitely. This strategy would be a complete waste of resources and would provide no profit, but rather a loss due to cleanup after the fish kill, for the park.



Dear Mr Tinn,

My name is Nicky, and I come from the Marine Rights and Protection Group Victoria (MRPGV). I have been studying and analysing the 4 strategies that you have considered for your trout farm. Through graphs, tables and my calculations in this letter, I wish to inform you of the strengths and weaknesses of the different strategies. I will also recommend to you the strategy that I think is the best option and tell you why we at MRPGV believe it is the right choice.

**MRPGV aims to: protect and respect marine life, giving long-term solutions that will benefit marine animals for the future.**

Strategy 1: In this strategy, nothing is done to interfere with the normal breeding of the trout. The Trout population keeps multiplying until the number reaches 5 times the original capacity, which in this case is 50000 trout (10000 x 5). At this point, up to 95% of the fish may die because of overpopulation. Between 8 and 9 years, the trout population reaches 50000 trout. The great thing

about this strategy is that the trout are left to multiply naturally, and we do think this is a great way for animals to live- without restraints. We do not know how long you would like this strategy to go for, but we know that the problem comes after 8 years. We believe in rights for marine animals, and if 95% of these fish die, we are not doing them justice at all. Hence, this strategy fails after 8 years. We want to let these trout live and breed naturally, but not let them die after 8 years.

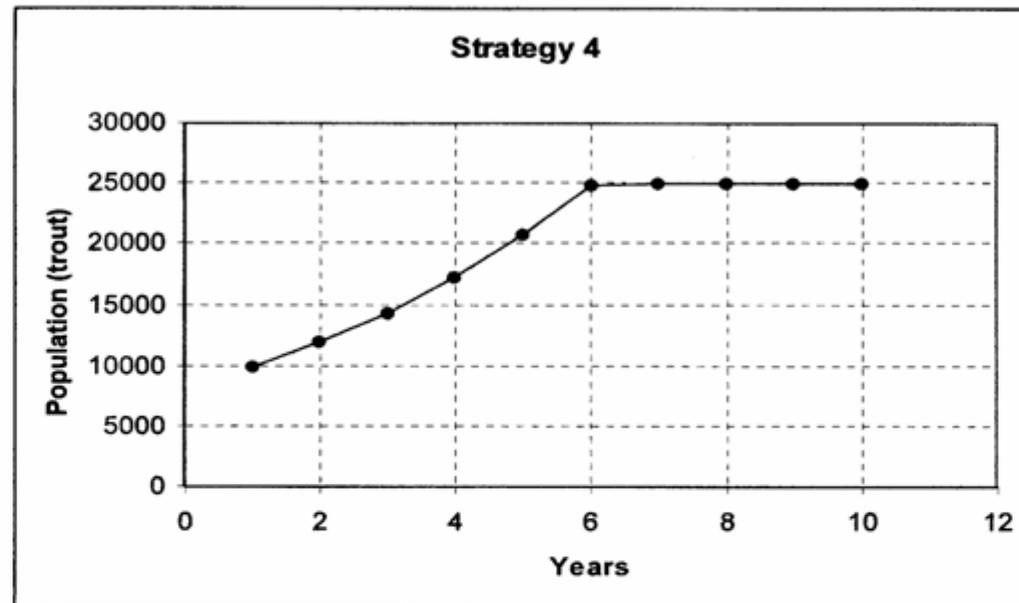
Strategy 2: This strategy involves fishing licences to be issued that would have a limit of 1800 fish per year. The trout population still rises, but not at such a pace as strategy 1. It also goes for a long time. Again, we do not know how long you would like this strategy to go for, but we know that in between 20 and 21 years, 95% of the fish will die. This is not respecting the trout, as we know that they will die in 20 odd years. This is a good option depending on how long this trout farm will prevail for.

Strategy 3: In this strategy, an open season is allowed in which up to 2500 fish could be removed from the lake each season. The trout population decreases, and after 8 years, there are no fish left, and there needs to be at least 750 fish in the lake for the fishery to remain viable. I believe that this

strategy defeats the purpose of having a successful trout farm, because it only does good for trout fishers who want trout. The trout are not thought about, and in the end, they are all gone because of the trout fishers. The rights of the trout are not thought of in this strategy, so I believe this is not a beneficial strategy.

Strategy 4: This strategy lets the trout grow naturally until it exceeds 25000 fish, and then fishing licences are issued that would maintain fish stock at this level. This option starts off like strategy 1, but after 5 years it stays at the same level. The advantage of this strategy is that for however long

I have decided on a way to manage your latest investment, the trout farm. Strategy 4 would be the best, although, it will be hard work for the first few years. Here is my analysis of the Strategy and what we need to do: We are aiming to have the farm, a place for people to fish, and with plenty of trout to go round. By keeping the level of trout at 25000, then we are sure to have enough  $\rho$  at all times.



I want to run my farm by having people pay for the weight of the fish, not fishing licences.

Costs:

Fresh Trout \$8 each

I will charge people \$2 entry fee each, plus for each fish they catch and keep. My yearly income will be:  
5000 trout x \$8.

+ entry fees.

Just the trout costs, will come to a total of \$40,000 per year.

Any fish not caught (not the full 5000) will be netted and sold at the market.

With entry fees, I could add on another few thousand.

For first few years I will make no profit, so instead of the fishing, I will make it into a nature reserve, with \$3 entry per person. You will not make much money, but the next years will make up for it.

*Today.*

Appropriate  
Content

Develop Deep  
Understanding

Motivate to take  
Risks

Active  
participation

Investigate  
the Real  
World

*Wild*

Time

*The Teaching Puzzle*

*Letting them know themselves as thinkers*



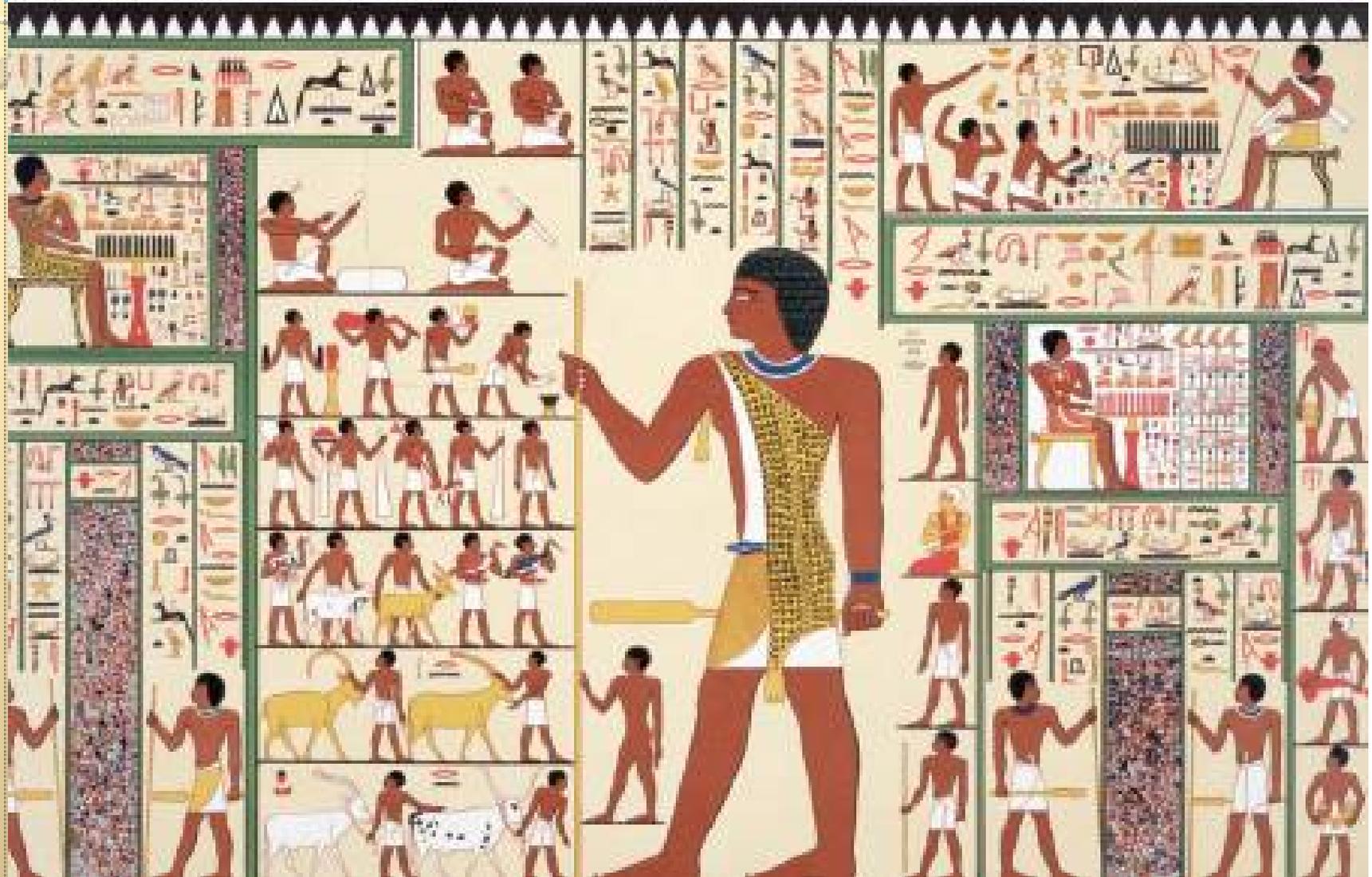
The best time to plant a tree  
is

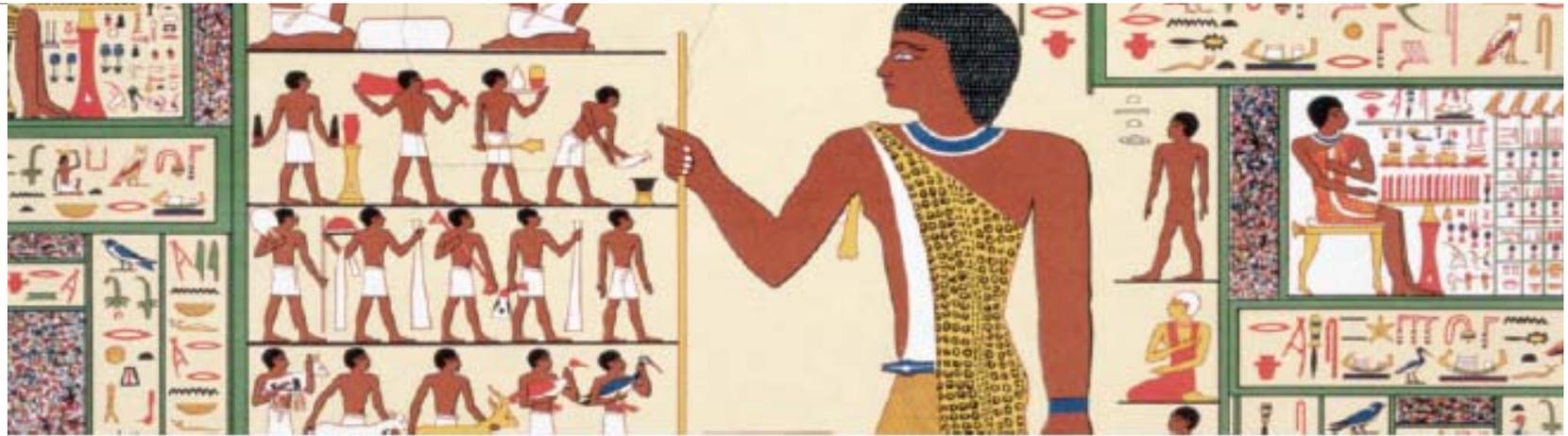
**twenty years ago;**

the second best time is

*today.*

# *Year 7 Mathematics*





This chapter covers the following skills:

- Understanding place value
- Adding, subtracting, multiplying and dividing whole numbers
- Doubling and halving mentally
- Estimating values
- Converting numbers from different counting systems
- Simplifying using order of operations
  - B Work out the calculations inside the brackets first. If there is more than one operation inside the brackets, then they must also follow the rules of BODMAS
  - O If the question contains fractions of or powers of, then these are calculated next
  - D Work out the division and
  - M multiplication calculations, working across the page from left to right
  - A Work out the addition and
  - S subtraction calculations, working across the page from left to right

**NUMBER**



**This chapter covers the following skills:**

- Exploring and comparing number patterns
- Finding factors, common factors and highest common factors of given numbers
- Finding multiples, common multiples and lowest common multiples of given numbers
- Expressing a number as a product of its prime factors
- Revising divisibility tests
- Exploring prime and composite numbers
- Expressing products of factors in index form
- Finding squares and square roots
- Investigating odd and even numbers

# Which square number ends in 2 ?

## NUMBER

- Write down the squares for the first 20 numbers.

$$1^2 =$$

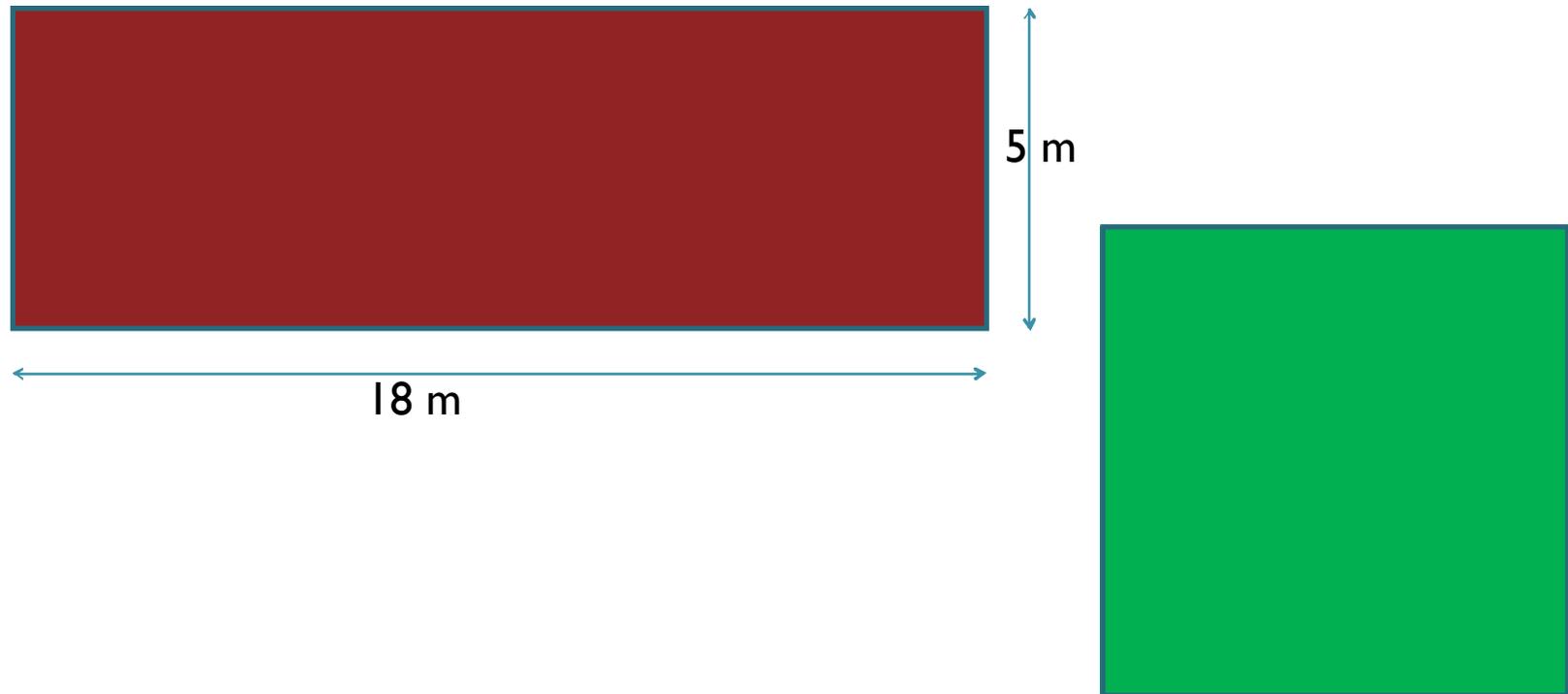
$$2^2 =$$

- Write out the squares for  
1, 11, 21, 31, 41, 51, 61, 71, 81,  
91

Observe the results. Make a general statement that could be always TRUE about the question

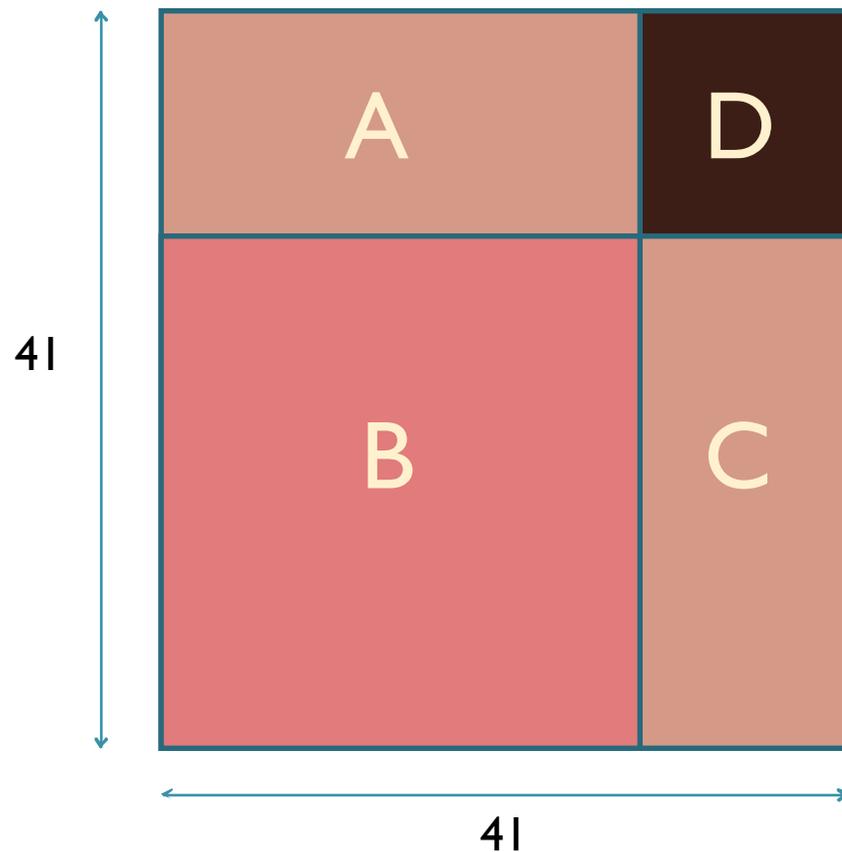
What is the length of the side of a square that has the same area as the rectangle?

**NUMBER**



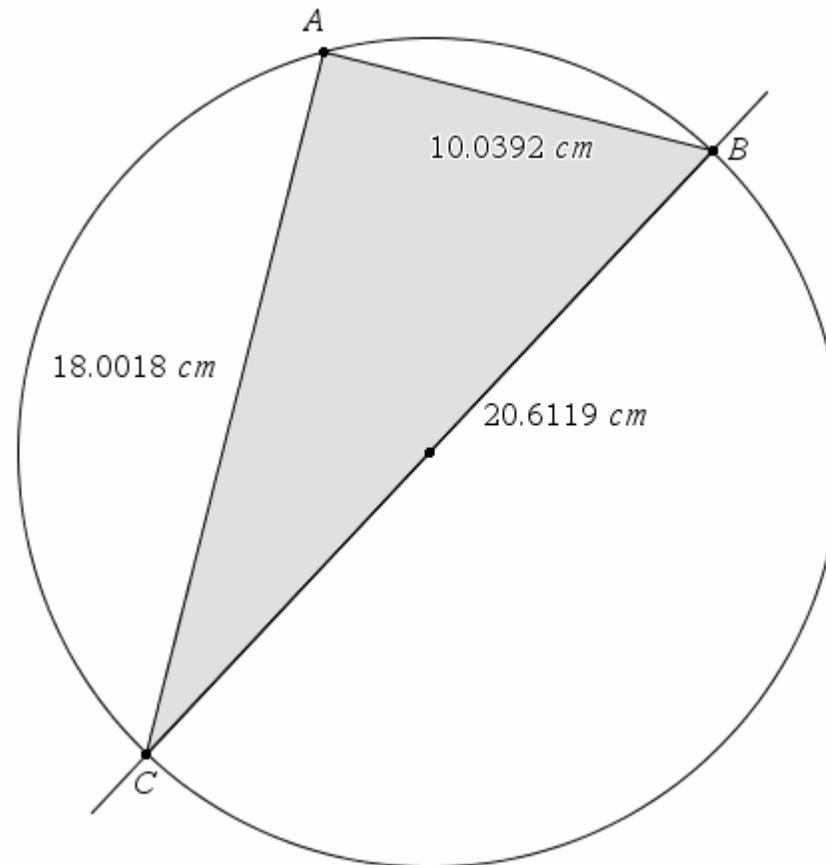
*How can you mentally calculate  $41^2$  ?*

**NUMBER**



The triangle in a circle where one side is a diameter for the circle.

**NUMBER**



# What number(s) divide into the sum of 3 consecutive whole numbers ?

## *NUMBER*

- Use 27 as the first of the 3 consecutive numbers
- How do you make consecutive whole numbers?
- What number divides into the sum of the 3 consecutive numbers, without having a remainder?
- Make another 3 consecutive numbers, and calculate the total of the 3 numbers.
- Do any of the numbers that divide into the first total number divide into the second total number?
- What have you found about the numbers that divide into the sum of 3 consecutive whole numbers?
- Test your ideas on the sum of 3 other consecutive numbers.
- If there is a pattern, can you suggest why it exists?

*Could this statement be true?*

**NUMBER**

5 is a factor of the sum of 5  
consecutive whole numbers but

4 is a factor of the sum of 4  
consecutive whole numbers



10 events for one medal THE DECATHLON

HOW DO THEY CONVERT  
THE TIMES AND  
DISTANCES FOR THE  
EVENTS in the DECATHLON.  
PENTATHLON and the  
Modern PENTATHLON into  
POINTS?

IS THE SCORING SYSTEM  
FAIR?

# FOLD PAPER



## *GEOMETRY*

Fold a piece of paper to produce

a Rectangle

a Square

a Parallelogram

a Kite

a Rhombus

a Right Angle Triangle

an Isosceles Triangle

an Equilateral Triangle



**This chapter covers the following skills:**

- Using the language of fractions
  - Simplifying and finding equivalent fractions
  - Converting improper fractions to mixed numbers and vice versa
  - Adding and subtracting with like and unlike denominators
  - Investigating, with a calculator, multiplication of fractions
  - Finding fractions of whole quantities
  - Dividing fractions
  - Finding squares and square roots of fractions
  - Simplifying using order of operations
- B** Work out the calculations inside the brackets first. If there is more than one operation inside the brackets, then they must also follow the rules of BODMAS
- O** If the question contains fractions of or powers of, then these are calculated next
- D** Work out the division and
- M** multiplication calculations, working across the page from left to right
- A** Work out the addition and
- S** subtraction calculations, working across the page from left to right



**This chapter covers the following skills:**

- Revising place value, notation and estimation of decimals
- Comparing decimals
- Rounding decimals
- Addition, subtraction, multiplication and division skills
- Comparing decimals to fractions and percentages
- Finding the percentage of a number
- Multiplying and dividing by powers of 10
- Calculating percentages